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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Use of statistical tables is permitted.***

PART – A

1 a. Using Taylor series method, solve the problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.2$. Consider upto 4th degree terms. **(06 Marks)**

b. Using R.K. method of order 4, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ at the points $x = 0.1$ and $x = 0.2$ by taking step length $h = 0.1$. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at $x = 0.8$ by Adams-Bashforth predictor-corrector method. Use the corrector formula twice. **(07 Marks)**

2 a. Evaluate y and z at $x = 0.1$ from the Picards second approximation to the solution of the following system of equations given by $y = 1$ and $z = 0.5$ at $x = 0$ initially.

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = x^3(y + z)$$ **(06 Marks)**

b. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ and $y'(0.2)$ by taking $h = 0.2$ and using fourth order Runge-Kutta method. **(07 Marks)**

c. Applying Milne's method compute $y(0.8)$. Given that y satisfies the equation $y'' = 2yy'$ and y and y' are governed by the following values. $y(0) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$, $y'(0) = 1$, $y'(0.2) = 1.041$, $y'(0.4) = 1.179$, $y'(0.6) = 1.468$. (Apply corrector only once). **(07 Marks)**

3 a. Derive Cauchy Riemann equations in Cartesian form. **(06 Marks)**

b. Find an analytic function $f(z) = u + iv$. Given $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$. **(07 Marks)**

c. If f(z) is a regular function of z, show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4\,|f'(z)|^2$ **(07 Marks)**

4 a. Find the bilinear transformation that maps the points $z = -1, i, -1$ onto the points $w = 1, i, -1$ respectively. **(06 Marks)**

b. Find the region in the w-plane bounded by the lines $x = 1$, $y = 1$, $x + y = 1$ under the transformation $w = z^2$. Indicate the region with sketches. **(07 Marks)**

c. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)}dz$ where c is the circle $|z| = 3$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Solve the Laplaces equation in cylindrical polar coordinate system leading to Bessel differential equation. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express the polynomial, $2x^3 - x^2 - 3x + 2$ interms of Legendre polynomials. **(07 Marks)**

6 a. State and prove addition theorem of probability. **(06 Marks)**

b. Three students A, B, C write an entrance examination. Their chances of passing are ½, ⅓, ¼ respectively. Find the probability that,
   i) Atleast one of them passes.
   ii) All of them passes.
   iii) Atleast two of them passes. **(07 Marks)**

c. Three machines A, B, C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective outputs of these three machines are respectively 2%, 3% and 4%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C. **(07 Marks)**

7 a. The pdf of a random variable x is given by the following table:

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

Find: i) The value of k ii) $P(x > 1)$ iii) $P(-1 < x \leq 2)$
iv) Mean of x v) Standard deviation of x. **(06 Marks)**

b. In a certain factory turning out razar blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing, i) One defective, ii) Two defective, in a consignment of 10000 packets. **(07 Marks)**

c. In a normal distribution 31% of items are under 45 and 8% of items are over 64. Find the mean and standard deviation of the distribution. **(07 Marks)**

8 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39350 kilometers with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40000 kilometers? (Use 0.05 level of significance) Establish 99% confidence limits within which the mean life of tyres expected to lie. (Given that $Z_{0.05} = 1.96$, $Z_{0.01} = 2.58$) **(06 Marks)**

b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given that $t_{0.05} = 2.262$ for 9 d.f.) **(07 Marks)**

c. Fit a Poisson distribution to the following data and test the goodness of fit at 5% level of significance. Given that $\psi^2_{0.05} = 7.815$ for 4 degrees of freedom.

x	0	1	2	3	4
Frequency	122	60	15	2	1

(07 Marks)

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10CV/CT42

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016
Concrete Technology

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Use of IS 10262-2009 is permitted.***

PART – A

1 a. Explain with flow chart, the cement manufactured by wet process. **(10 Marks)**
b. List the different laboratory tests conducted on cement. Explain any one of them in detail. **(10 Marks)**

2 a. Explain the role/effect of fine and coarse aggregates in concrete. **(10 Marks)**
b. Write a note on "mechanical properties" of coarse aggregates. Explain the test for determination of aggregate impact value as per IS2386 part IV – 1903. **(10 Marks)**

3 a. Define the term "Workability." List the factors affecting workability of concrete and methods of measurement of workability. **(10 Marks)**
b. List the various stages/process of production of quality concrete. Explain in brief. **(10 Marks)**

4 a. Discuss the role of chemical admixtures and mineral admixtures in cement concrete. **(10 Marks)**
b. Write short notes on: (i) Rice husk ash, (ii) Air entering agents **(10 Marks)**

PART – B

5 a. List the factors influencing strength of concrete. **(06 Marks)**
b. What is the relation between:
i) Compressive strength and tensile strength of concrete as per IS456-2000
ii) Cube strength and cylindrical strength of concrete. **(04 Marks)**
c. Explain in brief the principles of flexural/modulus of rupture testing of concrete under third-point loading method. **(10 Marks)**

6 a. Mention the different modulii of elasticity of concrete. **(04 Marks)**
b. What are the factors affecting shrinkage and creep of concrete? **(06 Marks)**
c. Write short notes on:
i) plastic shrinkage
ii) drying shrinkage **(10 Marks)**

7 a. Define the term durability. What is its significance and its impact on W/C ratio? **(10 Marks)**
b. Write short notes on:
i) Sulphate attack and its control
ii) Freezing and thawing and its remedial measures **(10 Marks)**

8 a. Discuss the steps involved in Indian standard method of concrete mix design (IS 10262-1982). **(12 Marks)**
b. List out the variables in proportioning of concrete mix design. **(08 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016
Structural Analysis – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define : (i) Degree of static indeterminacy and ii) Degree of freedom. **(05 Marks)**

b. Derive an expression for strain energy due to bending. **(06 Marks)**

c. Determine the degree of static and kinematic indeterminacies for the structures shown in Fig.Q1(c) **(09 Marks)**

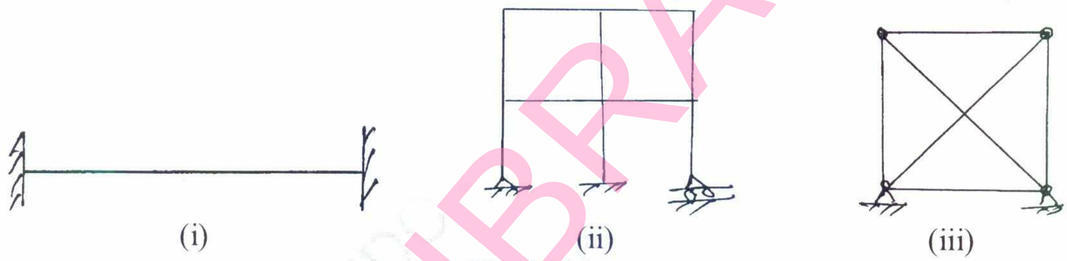

Fig.Q1(c)

2 a. For the beam loaded as shown in Fig.Q2(a), determine the slope and deflection at the free end. Take $E = 204\times10^6$ kN/m^2. Use moment area method. **(10 Marks)**

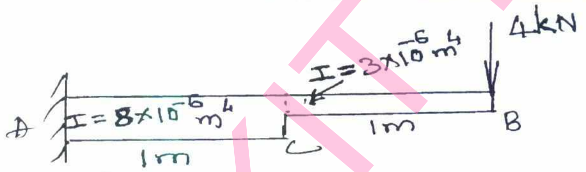

Fig.Q2(a)

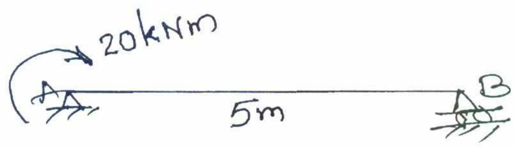

Fig.Q2(b)

b. Calculate the maximum slope and maximum deflection for the beam shown in Fig.Q2(b). Use conjugate beam method. Take $E = 2\times10^5$ N/mm^2, $I = 40\times10^6$ mm^4. **(10 Marks)**

3 a. State Castigliano's First and second theorems. **(04 Marks)**

b. Using Castigliano's theorem compute the deflection at the mid-point of simply supported beam of span 'L' and flexural rigidity EI, carrying a point load 'W' at the midspan. **(06 Marks)**

c. Determine the vertical and horizontal deflection at 'C' of the beam shown in Fig.Q3(c). Take E = 200 GPa and $I = 80\times10^6$ mm^4. **(10 Marks)**

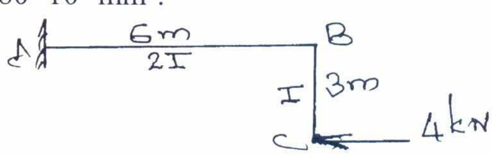

Fig.Q3(c)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. Using strain energy method determine reaction at B for the beam shown in Fig.Q4(a). Draw BMD and SFD. **(10 Marks)**

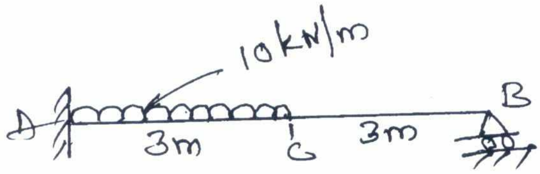

Fig.Q4(a)

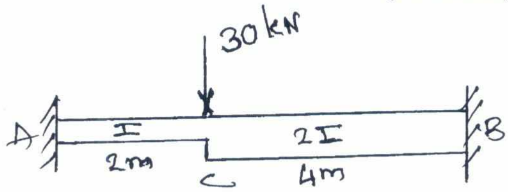

Fig.Q4(b)

b. Analyse the fixed beam shown in Fig.Q4(b) using strain energy method and draw BMD. **(10 Marks)**

PART – B

5 a. A three hinged symmetrical parabolic arch has a span of 36m and central rise of 6m. The arch carries a UDL of intensity 30 kN/m over left half portion and a concentrated load of 60 kN at 9 m from right support. Compute the bending moment, normal thrust and radial shear at 9 m from left support. **(12 Marks)**

b. A cable is suspended between two points A and B, 80 m apart horizontally and a central dip of 6 m. It supports a UDL of intensity 20 kN/m. Compute:

i) The length of the cables.

ii) Maximum and minimum tension in the cable. **(08 Marks)**

6 a. For the beam shown in Fig.Q6(a), compute the reaction at B by consistent deformation method. Draw BMD and SFD. **(10 Marks)**

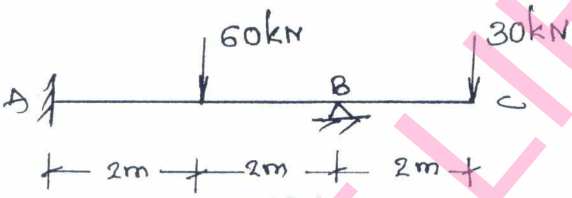

Fig.Q6(a)

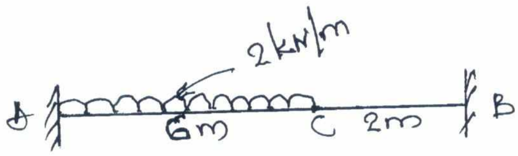

Fig.Q6(b)

b. Analyse the fixed beam shown in Fig.Q6(b) using consistent deformation method. **(10 Marks)**

7 Analyse the continuous beam shown in Fig.Q7 by Clapeyron's theorem. Draw SFD & BMD. **(20 Marks)**

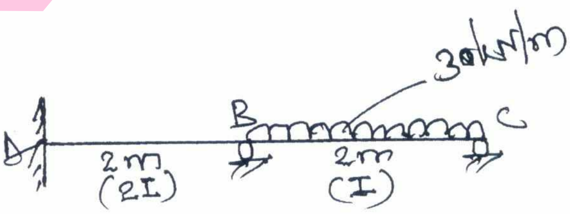

Fig.Q7

8 A two hinged parabolic arch has a span of 32m and a rise of 8m. A uniformly distributed load of 1 kN/m covers 8 m horizontal length of the left side of the arch. If $I = I_0 \sec\theta$, where θ is the inclination of the arch section to the horizontal, and I_0 is the moment of inertia at the crown. Find the horizontal thrust at hinged and bending moment at 8 m from the left hinge. Also find the normal thrust and radial shear at this section. **(20 Marks)**

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10CV44

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016
Surveying – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Distinguish between:
   i) Plunging and swinging of the telescope.
   ii) Clamp screw and tangent screw.
   iii) Transitting and line of collimation.
   iv) Plate bubble and altitude bubble. **(08 Marks)**

b. With neat sketches, explain measurement of a straight line by a theodolite in adjustment and theodolite not in adjustment. **(08 Marks)**

c. List the errors eliminated by repetition method. **(04 Marks)**

2 a. What are the permanent adjustments of a theodolite? Explain the spire test. **(08 Marks)**

b. Explain the procedure to lay off an angle with a greater precision than the least count of the instrument. **(08 Marks)**

c. What is an 'index error'? Why it is necessary to be adjusted? **(04 Marks)**

3 a. Derive the expression for the horizontal distance, vertical distance and the elevation of an elevated object by double plane method, when the base is inaccessible. **(07 Marks)**

b. List the advantages of total station over the conventional surveying instruments. **(03 Marks)**

c. In order to ascertain the elevation of the top (Q) of the signal on a hill, observations were made from two instrument stations P and R at a horizontal distance 100 M apart, the stations P and R being line with Q. The angles of elevation of Q at P and R were $28^\circ 42'$ and $18^\circ 6'$ respectively. The staff readings upon the bench mark of elevation 287.280 M were respectively 2.870 M and 3.750 M when the instrument was at P and R, the telescope being horizontal. Determine the elevation of the foot of the signal if the height of the signal above its base is 3 M. **(10 Marks)**

4 a. Write a note on:
   i) Anallactic lens. ii) Subtense diaphragm. **(04 Marks)**

b. Explain the method of determining the constant of a tachometer, in the field. **(06 Marks)**

c. A tachometer is setup at an intermediate point on a traverse course PQ and the following observations were made on a vertically held staff.

Staff station	Vertical angle	Staff intercept (M)	Axial Hair Reading (M)
P	$+8^\circ 36'$	2.350	2.105
Q	$+6^\circ 6'$	2.055	1.895

The instrument is fitted with anallactic lens and the constant is 100. Compute the length of PQ and reduced level of Q, that of P being 321.500 M. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Define degree of a curve. Establish the relationship between degree of curve and its radius. **(05 Marks)**

b. Calculate the ordinates at 10 M for a circular curve, given that the length of long chord is 80 M and the versed sine is 4 M. **(05 Marks)**

c. The chainage at the point of intersection of the tangents to a railway curve is 1250 M and the angle between them is 150°. Calculate all the data necessary for curve setting out a curve of radius 250 M by the deflection angle method. The peg intervals may be taken as 20 M. Also apply the arithmetic check. **(10 Marks)**

6 a. Distinguish between compound curve and reverse curve. **(04 Marks)**

b. Derive the relationships between various elements of a reverse curve between two parallel straights. **(08 Marks)**

c. Two parallel railway lines are to be connected by a reverse curve, each section having the same radius. If the lines are 12 M apart and the maximum distance between tangent points measured parallel to the straights is 48 M, find the maximum allowable radius. If both the radii are to be different, calculate the radius of the second branch if that of the first branch is 60 M. Also calculate the length of both the branches. **(08 Marks)**

7 a. What is a transition curve? Explain the requirements of a transition curve. **(06 Marks)**

b. Why are vertical curves provided on highways? List the different types of vertical curves. **(04 Marks)**

c. A transition curve is required for a circular curve of 200 M radius, the gauge being 1.5 M and maximum super elevation is restricted to 0.15 M. The transition is designed for a velocity such that no lateral pressure is imposed on the rails and the rate of gain of radial acceleration is 30 cm/sec^3. Calculate the required length of curve and the design speed. **(10 Marks)**

8 a. What is "Zero circle" of a planimeter? Explain any one method of finding its area. **(06 Marks)**

b. Calculate the area of a figure from the following using a planimeter with the point inside the figure.
Initial reading = 9.918
Final reading = 4.254
Constant M and C respectively are 100 cm^2 and 23.521. The zero mark of the disc crossed the index once in anticlockwise direction. **(04 Marks)**

c. The areas enclosed by the contours in a lake are as follows:

Contour (M)	270	275	280	285	290
Area (M^2)	2050	8400	16300	24600	31500

Calculate the volume of water between the contour 270 M and 290 M by the trapezoidal and prismoidal rule. **(10 Marks)**

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10CV45

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Hydraulics and Hydraulic Machines

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Missing data may suitably be assumed.

PART – A

1 a. Define the dimensional homogeneity. Give an example. **(06 Marks)**

b. Briefly explain geometric, kinematic and dynamic similarities. **(09 Marks)**

c. A 2.5m ship model was tested in fresh water $\rho = 1000 kg/m^3$ and measurements indicated that there was resistance of 45N when the model was moved at 2m/s. Workout the velocity of 40m prototype. Also calculate the force required to drive the prototype at this speed through sea water ($\rho = 1025 kg/m^3$). **(05 Marks)**

2 a. With neat sketches differentiate between flow through pipes and flow through open channels with examples. **(08 Marks)**

b. Derive an expression for the discharge through an open channel using Manning's formula. **(06 Marks)**

c. An earthen channel with a base width 2m and side slope 1H to 2V carries water with a depth of 1m. The bed slope is 1 in 625. Calculate the discharge if n = 0.03. Also calculate average shear stress at the channel boundary. **(06 Marks)**

3 a. Define specific energy. Draw specific energy curve, and then derive expressions for critical depth, critical velocity and minimum specific energy. **(10 Marks)**

b. Derive the expression for sequent depth of hydraulic jump interms of Froude number before a hydraulic jump in a rectangular channel flow. **(06 Marks)**

c. A horizontal rectangular channel 4m wide carries a discharge of $16 m^3/s$. Determine whether a jump may occur at an initial depth of 0.5m or not. If a jump occurs, determine the sequent detpth to this initial depth. Also determine the energy loss in the Jump. **(04 Marks)**

4 a. State impulse momentum equation. **(02 Marks)**

b. Show that in case of jet striking the flat plates mounted on wheels, the efficiency will be maximum when the tangential velocity of wheel is half that of jet and maximum efficiency is only 50%. **(10 Marks)**

c. A 75mm diameter jet having a velocity of 30m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal force exerted on the plate.
i) When the plate is stationary ii) when plate is moving with velocity of 15m/s in the direction of jet away from the jet.
Also determine the power and efficiency of the system when the plate is moving. **(08 Marks)**

PART – B

5 a. Show that when the jet of water striking symmetrical moving curved vane at the centre, the maximum efficiency for semicircular vane is $\frac{8}{27}(1+\cos\theta)$. **(10 Marks)**

b. A jet of water, 60mm in diameter, strikes a curved vane at its centre with a velocity of 18m/s. The curved vane is moving with a velocity of 6m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate to be smooth, Find :
   i) Thrust on the plate in the direction of jet
   ii) Power of the jet, and
   iii) Efficiency of the jet **(10 Marks)**

6 a. With the help of velocity triangles derive an expression for work done and maximum hydraulic efficiency of a pelton wheel. **(10 Marks)**

b. A Pelton wheel is receiving water from a penstock with a gross head of 510m. One third of gross head is lost in friction in the penstock. The rate of flow through the nozzle fitted at the end of the penstock is $2.2m^3/s$. the angle of deflection of the jet is 165°. Determine :
   i) Power given by water to the runner,
   ii) Hydraulic efficiency of the pelton wheel.
   Take $C_V = 1.0$ and speed ratio = 0.45. **(10 Marks)**

7 a. Draw the neat sketch of Kaplan turbine and mention the parts. **(08 Marks)**

b. Explain with the help of neat sketches the different types of draft tubes. **(06 Marks)**

c. A Kaplan turbine develops 22000kW at an average head of 35m. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner and an overall efficiency of 88%, calculate the diameter, speed and specific speed of the turbine. **(06 Marks)**

8 a. Explain briefly the various types of efficiencies of a centrifugal pump. **(08 Marks)**

b. Distinguish between pumps in series and pumps in parallel. **(07 Marks)**

c. A centrifugal pump is to discharge $0.118m^3/s$ at a speed of 1450rpm against a head of 25m. The impeller diameter is 250mm, its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller. **(05 Marks)**

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10CV46

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016
Building Planning and Drawing

Time: 4 hrs. Max. Marks:100

Note: ***1. PART – A is compulsory.***
2. Answer any TWO full questions from PART – B.

PART – A

1 Draw the plan, elevation and sectional elevation for the line diagram of the building shown in Fig. Q1. Also write the schedule of openings.
a. Plan of the building. **(25 Marks)**
b. Elevation. **(15 Marks)**
c. Section at A – A. **(15 Marks)**
d. Schedule of openings. **(05 Marks)**

PART – B

2 Draw to a suitable scale, the plan, elevation and vertical section of a six paneled double leaf door with overall size of 1.2 m × 2.1 m. **(20 Marks)**

3 Prepare a bubble diagram (connectivity diagram) and develop a line diagram for the primary health centre to a suitable scale. The following particulars may be assumed suitably.
i) Waiting hall
ii) Office
iii) Doctor's room
iv) Technician's room
v) Lady doctor's room
vi) Dispensary
vii) Store
viii) Nurse's room
ix) 2 numbers of general toilet, one for gents and other for ladies (suitable)
x) Generous verandah. **(20 Marks)**

4 Prepare a bubble diagram (connectivity diagram) and develop a line diagram for primary school building with the following requirements. The strength of the school is 400 students.
i) Class room 8 numbers
ii) Head master's room with attached toilet 1 number
iii) Staff room 1 number
iv) Office room 1 number
v) Library 1 number
vi) Auditorium 1 number
vii) Toilets to be provided separately for boys and girls. **(20 Marks)**

5 The line diagram of a residential building is shown in Fig. Q5. Prepare water supply and sanitary layout plans with usual notations to a scale 1 : 50. **(20 Marks)**

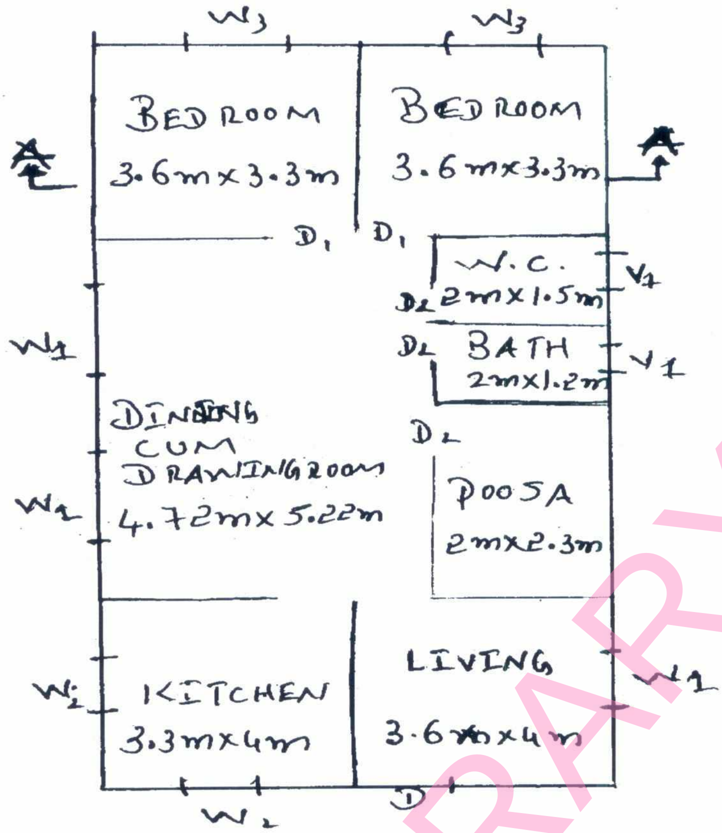
W3
W3
BED ROOM
3.6m x 3.3m
BED ROOM
3.6m x 3.3m
D1
D1
W.C.
2m x 1.5m
V1
D2
BATH
2m x 1.2m
V1
W1
DINING CUM DRAWING ROOM
4.72m x 5.22m
W1
D2
POOSA
2m x 2.3m
W1
KITCHEN
3.3m x 4m
LIVING
3.6m x 4m
W1
W2
D

Fig. Q1

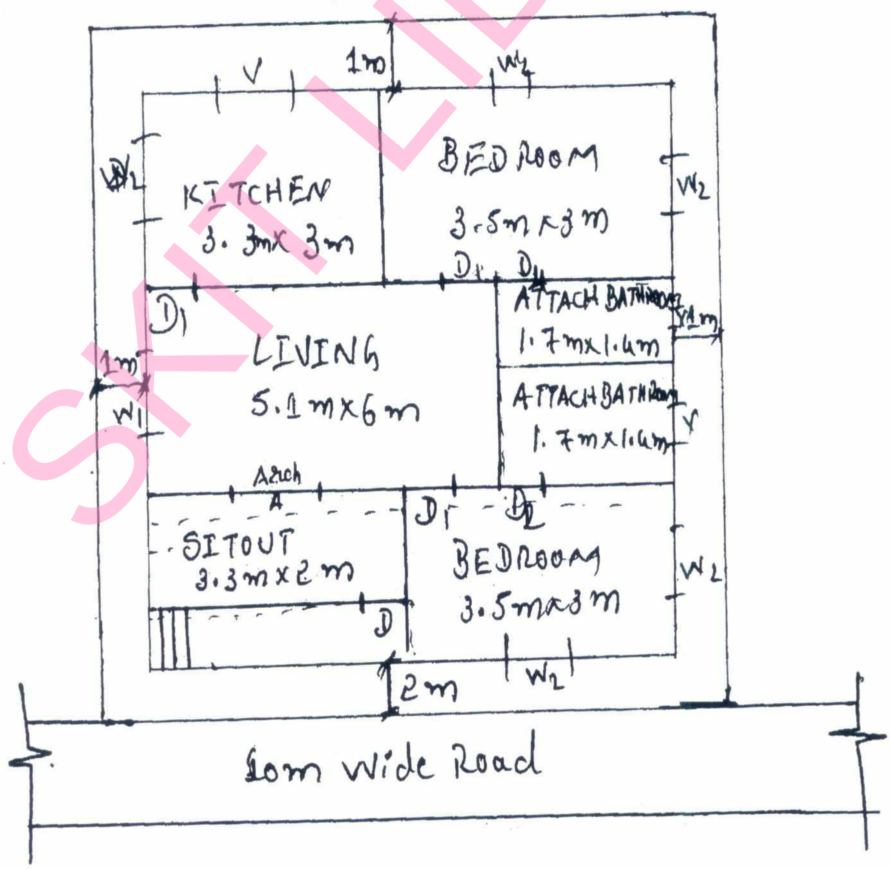
V
1m
W1
W2
KITCHEN
3.3m x 3m
BED ROOM
3.5m x 3m
W2
D1
D1
D1
ATTACH BATHROOM
1.7m x 1.4m
1m
LIVING
5.1m x 6m
1m
W1
ATTACH BATHROOM
1.7m x 1.4m
V
Arch
A
D1
D2
SITOUT
3.3m x 2m
BEDROOM
3.5m x 3m
W2
D
2m
W2
10m Wide Road

Fig. Q5

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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines (3, -1, 1) an (-3, 2, 4). **(06 Marks)**

b. If cos α, cos β, cos γ are the direction cosines of a line, then prove the following:

i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ **(07 Marks)**

c. Find the projection of the line AB on the line CD where A = (1, 2, 3), B = (1, 1, 1), C = (0, 0, 1), D = (2, 3, 0). **(07 Marks)**

2 a. Find the equation of the plane through (1, -2, 2), (-3, 1, -2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

b. Find the image of the point (1, -2, 3) in the plane 2x + y – z = 5. **(07 Marks)**

c. Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **(07 Marks)**

3 a. Find the constant 'a' so that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. **(06 Marks)**

b. Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$. **(07 Marks)**

c. Find the unit normal vector to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. **(07 Marks)**

4 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, z = 2t + 5 where t is the time. Find the components of its velocity and acceleration at time t = 1 in the direction of $2i + 3j + 6k$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at the point (2, -1, 2). **(07 Marks)**

c. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (1, -2, -1) in the direction of the normal to the surface $x \log z - y^2 = -4$ at (-1, 2, 1). **(07 Marks)**

5 a. Prove that $\operatorname{div}(\operatorname{curl} \vec{A}) = 0$. **(06 Marks)**

b. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Show that the vector $\vec{F} = (3x^2 - 2yz)i + (3y^2 - 2zx)j + (3z^2 - 2xy)k$ is irrotational and find ϕ such that $\vec{F} = \operatorname{grad}\phi$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator, will be treated as malpractice.

6 a. Find: $L\{\cos t \; \cos 2t \; \cos 3t\}$. **(06 Marks)**

b. Find: i) $L\{e^{-t}\cos^2 t\}$, ii) $L\{te^{-t}\sin 3t\}$. **(07 Marks)**

c. Find: $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$. **(07 Marks)**

7 a. Find: $L^{-1}\left\{\dfrac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

b. Find: i) $L^{-1}\left\{\dfrac{s+2}{s^2-4s+13}\right\}$, ii) $L^{-1}\left\{\log\left(\dfrac{s+a}{s+b}\right)\right\}$. **(07 Marks)**

c. Find: $L^{-1}\left\{\dfrac{1}{s^2(s+1)}\right\}$. **(07 Marks)**

8 a. Using Laplace transforms, solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^{2t}$ with $y(0)=0$, $y'(0)=1$. **(10 Marks)**

b. Using Laplace transformation method solve the differential equation $y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$. **(10 Marks)**

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